Title: Blast Design - Equivalent Static Methods

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ABSTRACT

The blast pressure on a structure due to a nearby explosion is typically of very high magnitude and very short duration. Such an impulsive loading usually requires dynamic time-history analysis, even when affected elements are simplified by using single-degree-of-freedom models. An alternative is to use an empirical equation published by Newmark in 1956 that provides the required ultimate strength of an element given the peak blast force, the ratio of the positive phase duration to the natural period of vibration, and the target ductility ratio. Software is available to the public from the General Services Administration that can calculate the relevant properties of the blast load. Basic principles of structural dynamics provide the effective mass, stiffness, and strength of the resisting element. The Department of Defense has developed detailed response limits for various materials and element types that correspond to four levels of protection. Consequently, it is now possible to use an equivalent static approach for blast effects analysis and design, especially given the uncertainties and assumptions that are inherent even in more sophisticated procedures.

INTRODUCTION

Determining the effects of explosions on structures has historically been the exclusive domain of a handful of specialized firms and individuals, who have developed sophisticated tools and techniques for this purpose. However, various terrorist attacks in recent years have considerably increased the perceived risk of similar incidents, particularly at government, military, and diplomatic buildings, but also at certain private-sector facilities. Consequently, blast analysis is now becoming an integral part of the design process for these kinds of projects.

There are a number of uncertainties and assumptions that are inherent in blast effects analysis and design, even when complex dynamic loading and response models are employed. Typical examples include:

- Size and location of the explosive device.
- Magnitude and shape of the blast pressure history.
- Effects of ground shock, casing fragmentation, and secondary debris.
- Material properties and end conditions of affected elements.
- Correlation between element deflection and the degree of damage.
- Load transfer at connections to supporting elements.

Even minor variations in these parameters can have a significant impact on the predicted behavior. Recognizing this, it is appropriate under many circumstances to adopt a simple equivalent static methodology that is accessible to "ordinary" practitioners of structural engineering (Newmark 1956).

LOAD

The shock wave from an external explosion causes an almost instantaneous increase in pressure on nearby objects to a maximum value. This is followed by a brief positive phase, during which the pressure decays back to its ambient value, and a somewhat longer but much less intense negative phase, during which the pressure reverses direction. For most structures, this phenomenon can be approximated using a triangular impulse load with zero or minimal rise time and linear decay, as shown in Figure 1. This equivalent load is calibrated to match the maximum reflected pressure (pr) and total reflected impulse (ir) of the actual load's positive phase, so that the design duration td = 2ir/pr. The negative phase is neglected because it usually has little effect on the maximum response.

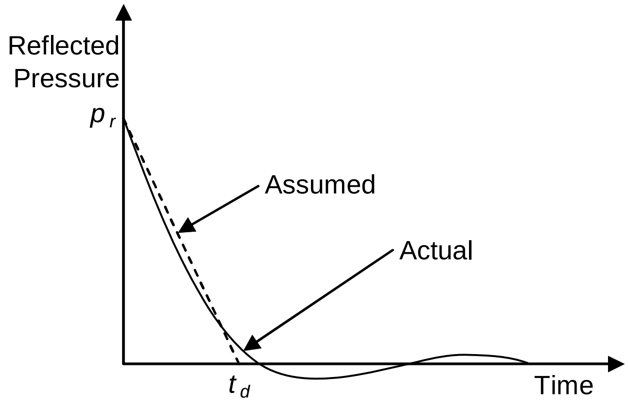

Figure 1. Blast Load on Structures

The values of pr and ir depend on the standoff distance (R) and explosive charge size (W), which are combined to determine the scaled distance parameter Z = R / W1/3. R measures how close to the building a bomb could explode and therefore depends on the physical characteristics of the surrounding site (Department of Defense 2007). W is expressed in weight or mass of TNT in order to correlate with tests; the equivalent W of any other explosive material is based on experimentally determined factors or the ratio of its heat of detonation to that of TNT (ASCE 1999). The angle of incidence at which a blast wave strikes the loaded surface also influences these parameters. A computer program that can perform these calculations, AT Blast, is available for downloading free of charge from the U. S. General Services Administration (www.oca.gsa.gov).

Although the actual blast load on an exposed element will vary over its tributary area, for design the maximum dynamic load (Fo) is typically taken as the product of this area and either the maximum pr or a spatially averaged value. This is analogous to the manner in which design wind loads for components and cladding are routinely calculated (ASCE 2005), and is generally valid when Z ≥ 3.0 ft/lb1/3. Blast loads need not be factored since they already represent an ultimate design condition.

RESISTANCE

An element loaded by a blast can be modeled approximately as an elastic-plastic dynamic system with a single degree of freedom (SDOF) corresponding to its maximum blast deflection (ymax). The element's effective mass (me), elastic and elastic-plastic stiffnesses (k1 and k2), and remaining yield and ultimate strengths after other loads have been applied (Ry and Rn) are derived from its actual physical configuration and properties (Biggs 1964). Table 1 summarizes these parameters for a uniformly loaded one-way element of any material with various end conditions in terms of its uniformly distributed mass (m), elastic modulus (E), moment of inertia (I), span length (L), and remaining nominal moment capacities at mid-span (Mnrm) and at fixed ends (Mnre). These values must obviously correspond to the axis of the bending induced in the element by the blast load. Similar tables exist for two-way elements with different combinations of edge conditions and aspect ratios (Departments of the Army, the Navy, and the Air Force 1990).

Table 1. Effective Properties of One-Way Elements

End Conditions	Mass m_e	Elastic Stiffness k_1	Ratio M_{nam}/M_{nae}	Elastic-Plastic Stiffness k_2	Yield Strength R_y	Ultimate Strength R_n
Fixed/Fixed	$0.78m$	$384EI/L^3$	$>1/2$	$76.8EI/L^3$	$12M_{nae}/L$	$8(M_{nam}+M_{nae})/L$
			$\leq 1/2$	$128EI/L^3$	$24M_{nam}/L$	$8(M_{nam}+M_{nae})/L$
Fixed/Pinned	$0.78m$	$185EI/L^3$	$>9/16$	$76.8EI/L^3$	$8M_{nae}/L$	$4(2M_{nam}+M_{nae})/L$
			$\leq 9/16$	$32.8EI/L^3$	$14.2M_{nam}/L$	$3.2(2.67M_{nam}+M_{nae})/L$
Fixed/Free	$0.65m$	$8EI/L^3$	N/A	0	$2M_{nae}/L$	$2M_{nae}/L$
Pinned/Pinned	$0.78m$	$76.8EI/L^3$	N/A	0	$8M_{nam}/L$	$8M_{nam}/L$

When a particular element is continuously connected to an adjacent one, a portion of the latter's mass often can be added to the element's own. For example, the designer can include the mass of 20% of the wall on each side of an integral pilaster and the full tributary length of metal panels attached to girts. However, any stiffness contribution from adjacent elements should usually be neglected. When the mid-span and end moments of inertia are unequal, the designer should use the average value. The same is true of the remaining moment capacities at the two ends of a fixed/fixed element. For a reinforced concrete or masonry element, stiffness calculations should be based on the average of the gross and cracked section moments of inertia (Departments of the Army, the Navy, and the Air Force 1990).

As shown in Figure 2, the simplified resistance function of a fixed/fixed or fixed/pinned element is tri-linear (ASCE 1997). Deflection consistent with the elastic stiffness k1 occurs until initial plastic hinge formation at the yield capacity Ry, then the elastic-plastic stiffness k2 governs up to the ultimate capacity Rn. Although it is possible to analyze the element using this resistance function, a common simplification that sacrifices little accuracy is to use an equivalent elastic stiffness calibrated to provide the same area under the curve and thus the same energy dissipation:

$$k_e = \frac{k_2}{1-\left(1-\frac{k_2}{k_1}\right)\left(\frac{R_y}{R_n}\right)\left(2-\frac{R_y}{R_n}\right)} \tag{1}$$

When the mid-span and end moment capacities are equal, this provides ke = 307 EI/L3 for fixed/fixed elements and ke = 160 EI/L3 for fixed/pinned elements.

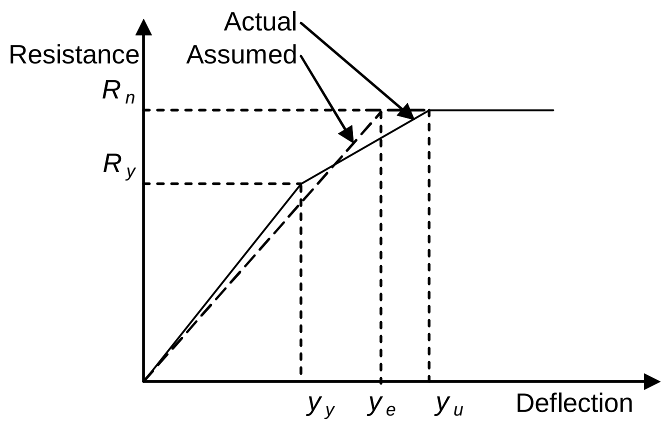

Figure 2. Resistance Function for One-Way Elements

Materials used in actual construction have strengths that exceed their specified minimum values by an average of roughly 10%. In addition, the short duration of a blast load results in high strain rates that increase the design strength further, depending on the material and failure mode as indicated in Table 2 (Departments of the Army, the Navy, and the Air Force 1990). The designer may also take advantage of the increase in concrete strength with age, which for ordinary Portland cement is on the order of 10% at six months and 15% at one year or more. This is reasonable since additional construction is almost always necessary after structural concrete has been placed before the building is occupied and becomes a potential target. Material-specific interaction equations account for the reduced moment capacities available to withstand a blast load because of the stresses already present due to the dead load and a realistic portion of the live and wind loads--usually on the order of 50% and 20%, respectively (ASCE 2005). All strength reduction or resistance factors are typically set to unity, because the objective is to predict the actual behavior of the element, rather than enforce a particular factor of safety against failure.

Table 2. Dynamic Increase Factors[a]

Material and Property	Failure Mode	DIF
Concrete Compressive Strength	Flexure	1.19
	Compression	1.12
	Direct Shear	1.10
Masonry Compressive Strength	Flexure	1.19
	Compression	1.12
	Direct Shear	1.10
Deformed Reinforcement Steel Yield Strength	Flexure	1.17
	Compression	1.10
	Direct Shear	1.10
	Bond	1.17
Welded Wire Reinforcement Steel Yield Strength	Flexure	1.10
Hot-Rolled Steel Yield Strength	Flexure/Shear	1.19[b]
	Tension/Compression	1.12[b]
Hot-Rolled Steel Ultimate Strength	All	1.05[b]
Cold-Formed Steel Yield Strength	Flexure/Shear	1.10
	Tension/Compression	1.10

[a]For materials, properties, and failure modes not listed, DIF = 1.00 unless another value is established using a rational procedure that accounts for strain rates and other relevant aspects of element response; assumed strain rates for this table are 0.10 in/in/sec for flexure and shear and 0.02 in/in/sec for tension and compression
[b]For ASTM A36 steel, it shall be permissible to use DIF = 1.29 for yield strength in flexure/shear, DIF = 1.19 for yield strength in tension/compression, and DIF = 1.10 for ultimate strength

An element that is subject to tension or high compression or that is slender requires a detailed investigation to determine Mnrm and Mnre, and thus Rn. For strong-axis bending of open-section structural or cold-formed steel elements, lateral bracing of the compression flange or torsional bracing of the cross section is required at plastic hinge locations and at a spacing small enough to preclude lateral-torsional buckling.

DESIGN

The response of an element to a triangular blast impulse depends on the following parameters:

- Ratio of blast impulse duration to natural period of vibration: td/T, .
- Ratio of maximum dynamic load to available ultimate strength: Fo/Rn.
- Ratio of maximum deflection to yield deflection: ductility ratio □ = ymax/ye = keymax/Rn.
- End rotation at supports: □ ≈ tan-1(2ymax/L), except □ ≈ tan-1(ymax/L) for cantilevers.

The ductility ratio and support rotation correlate with the expected amount of damage to an element in a blast event, which is restricted by the level of protection that the structure must provide to its occupants and contents based on their nature, quantity, function, and importance. Table 3 (Department of Defense 2007) describes four levels of protection, Table 4 (Protective Design Center 2006) indicates corresponding levels of damage to individual elements, and Tables 5 and 6 (Protective Design Center 2006) provide the accompanying □max and □max values for various materials and element types.

Table 3. Levels of Protection

Level of Protection	Potential Building Damage/Performance
Very Low	Heavy damage - Onset of structural collapse, but progressive collapse is unlikely. Space in and around damaged area will be unusable.
Low	Moderate damage - Building damage will not be economically repairable. Progressive collapse will not occur. Space in and around damaged area will be unusable.
Medium	Minor damage - Building damage will be economically repairable. Space in and around damaged area can be used and will be fully functional after cleanup and repairs.
High	Minimal damage. No permanent deformations. The facility will be immediately operable.

Table 4. Acceptable Damage

Level of Protection	Primary Structural Elements	Secondary Structural Elements	Non-structural Elements
Very Low	Heavy	Hazardous	Hazardous
Low	Moderate	Heavy	Heavy
Medium	Superficial	Moderate	Moderate
High	Superficial	Superficial	Superficial

In each case, it is possible to establish a single combined response limit for elements that are supported at both ends:

$$\mu_{cr} \approx \frac{k_e L}{2R_n} \tan\theta_{\max} \leq \mu_{\max} \tag{2}$$

For cantilevers, the factor of 2 is removed from this equation.

Analogous to the equivalent lateral force method for seismic design (ASCE 2002), the ultimate static design load for an element can be expressed as the maximum applied force divided by a response factor:

$$F_u = \frac{F_o}{R_b} \tag{3}$$

When the load duration is infinite, the element response is quasi-static, resulting in the following exact response factor for $\mu_{cr} \geq 1$ (Newmark 1956):

$$R_b = 1 - \frac{1}{2\mu_{cr}} \tag{4}$$

When the load duration is zero, the element response is impulsive, resulting in the following exact response factor for $\mu_{cr} \geq 1$ (Newmark 1956):

$$R_b = \frac{\sqrt{2\mu_{cr} - 1}}{\pi\left(t_d / T\right)} \tag{5}$$

Table 5. Response Limits for Flexural Elements[a]

Element Type	Expected Element Damage: Superficial μ_{max}	Superficial θ_{max}	Moderate μ_{max}	Moderate θ_{max}	Heavy μ_{max}	Heavy θ_{max}	Hazardous μ_{max}	Hazardous θ_{max}
Reinforced Concrete								
Single-Reinforced Slab or Beam	1	-	-	2°	-	5°	-	10°
Double-Reinforced Slab or Beam without Shear Reinforcement[b,c]	1	-	-	2°	-	5°	-	10°
Double-Reinforced Slab or Beam with Shear Reinforcement[b]	1	-	-	4°	-	6°	-	10°
Prestressed Concrete[d]								
Slab or Beam with $\omega_p > 0.30$	0.7	-	0.8	-	0.9	-	1	-
Slab or Beam with $0.15 \leq \omega_p \leq 0.30$	0.8	-	$0.25/\omega_p$	1°	$0.29/\omega_p$	1.5°	$0.33/\omega_p$	2°
Slab or Beam with $\omega_p \leq 0.15$ and without Shear Reinforcement[b,c]	0.8	-	$0.25/\omega_p$	1°	$0.29/\omega_p$	1.5°	$0.33/\omega_p$	2°
Slab or Beam with $\omega_p < 0.15$ and Shear Reinforcement[b]	1	-	-	1°	-	2°	-	3°
Masonry								
Unreinforced[c,e]	1	-	-	1.5°	-	4°	-	8°
Reinforced	1	-	-	2°	-	8°	-	15°
Structural Steel (Hot-Rolled)								
Beam with Compact Section[f]	1	-	3	3°	12	10°	25	20°
Beam with Noncompact Section[c,f]	0.7	-	0.85	3°	1	-	1.2	-
Plate Bent about Weak Axis	4	1°	8	2°	20	6°	40	12°
Open Web Steel Joist								
Downward Loading[g]	1	-	-	3°	-	6°	-	10°
Upward Loading[h]	1	-	1.5	-	2	-	3	-
Shear Response[i]	0.7	-	0.8	-	0.9	-	1	-
Cold-Formed Steel								
Girt or Purlin	1	-	-	3°	-	10°	-	20°
Stud with Sliding Connection at Top	0.5	-	0.8	-	0.9	-	1	-
Stud Connected at Top and Bottom[j]	0.5	-	1	-	2	-	3	-
Stud with Tension Membrane[k]	0.5	-	1	0.5°	2	2°	5	5°
Corrugated Panel (1-way)with Full Tension Membrane[l]	1	-	3	3°	6	6°	10	12°
Corrugated Panel (1-way) with Some Tension Membrane[m]	1	-	-	1°	-	4°	-	8°
Corrugated Panel (1-way) with Limited Tension Membrane[n]	1	-	1.8	1.3°	3	2°	6	4°
Wood[o]	1	-	2	-	3	-	4	-

[a]Where a dash (-) is shown, the corresponding parameter is not applicable as a response limit
[b]Stirrups or ties that satisfy sections 11.5.5 and 11.5.6 of ACI 318 and enclose both layers of flexural reinforcement throughout the span length
[c]These response limits are applicable for evaluation of existing elements only and shall not be used for design of new elements
[d]Reinforcement index $\omega_p = (A_{ps}/bd)(f_{ps}/f'_c)$
[e]Values assume wall resistance controlled by brittle flexural response or axial load arching with no plastic deformation; for load-bearing walls, use Superficial or Moderate damage limits to preclude collapse
[f]Limiting width-to-thickness ratios for compact and noncompact sections are defined in ANSI/AISC 360
[g]Values assume tension yielding of bottom chord with adequate bracing of top chord to prevent lateral buckling
[h]Values assume adequate anchorage to prevent pull-out failure and adequate bracing of bottom chord to prevent lateral buckling

[i]Applicable when element capacity is controlled by web members, web connections, or support connections; ductility ratio for shear is equal to peak shear force divided by shear capacity
[j]Also applicable when studs are continuous across a support
[k]Requires structural plate-and-angle bolted connections at top , bottom, and any intermediate supports
[l]Sheet has adequate connections to yield cross-section fully
[m]Typically applicable for simple-fixed span conditions
[n]Limited to connector capacity; includes all standing seam metal roof systems
[o]Values shown are based on very limited testing data; use specific test data if available

Table 6. Response Limits for Compression Elements[a]

	Expected Element Damage							
	Superficial		Moderate		Heavy		Hazardous	
Element Type	μ_{max}	θ_{max}	μ_{max}	θ_{max}	μ_{max}	θ_{max}	μ_{max}	θ_{max}
Reinforced Concrete								
Single-Reinforced Slab or Beam-Column	1	-	-	2°	-	2°	-	2°
Double-Reinforced Slab or Beam-Column without Shear Reinforcement[b,c]	1	-	-	2°	-	2°	-	2°
Double-Reinforced Slab or Beam-Column with Shear Reinforcement[b]	1	-	-	4°	-	4°	-	4°
Walls and Seismic Columns[d,e]	0.9	-	1	-	2	-	3	-
Non-seismic Columns[d,e]	0.7	-	0.8	-	0.9	-	1	-
Masonry								
Unreinforced[c,f]	1	-	-	1.5°	-	1.5°	-	1.5°
Reinforced	1	-	-	2°	-	2°	-	2°
Structural Steel (Hot-Rolled)								
Beam-Column with Compact Section[g,h]	1	-	3	3°	3	3°	3	3°
Beam-Column with Noncompact Section[g,h]	0.7	-	0.85	3°	0.85	3°	0.85	3°
Column (Axial Failure)[e]	0.9	-	1.3	-	2	-	3	-
Wood[i]								
Beam-Column (Flexural Failure)	1	-	2	-	2	-	2	-
Column (Axial Failure) [e]	-	-	-	-	-	-	1	2.4°

[a]Where a dash (-) is shown, the corresponding parameter is not applicable as a flexural response limit
[b]Stirrups or ties that satisfy sections 11.5.5 and 11.5.6 of ACI 318 and enclose both layers of flexural reinforcement throughout the span length
[c]These response limits are applicable for evaluation of existing elements only and shall not be used for design of new elements
[d]Seismic columns have ties or spirals that satisfy, at a minimum, the requirements of section 21.12.5 of ACI 318; see Chapter 9 for complete detailing requirements
[e]Ductility ratio is based on axial deformation, rather than flexural deformation
[f]Values assume wall resistance controlled by brittle flexural response or axial load arching with no plastic deformation; for load-bearing walls, use Superficial or Moderate damage limits to preclude collapse
[g]Limiting width-to-thickness ratios for compact and noncompact sections are defined in ANSI/AISC 360
[h]Use connection shear capacity, rather than element flexural capacity, to calculate ultimate resistance for analysis
[i]Values shown are based on very limited testing data; use specific test data if available

Equation (4) can generally be used for design when $t_d/T \geq 10$, while equation (5) is applicable when $t_d/T \leq 0.1$. Combining these and making minor empirical adjustments results in the following equation that has an error of 5% or less for all values of t_d/T and $\mu_{cr} \geq 1$ (Newmark 1956):

$$R_b \approx \frac{\sqrt{2\mu_{cr} - 1}}{\pi(t_d / T)} + \frac{(1 - 1/2\mu_{cr})(t_d / T)}{t_d / T + 0.7} \tag{6}$$

When $\mu > 1$, the element must actually be capable of undergoing the plastic deformation associated with its expected μ and θ values without suffering unacceptable damage. This requires careful detailing of members and especially connections. Although code requirements and industry guidelines for structures in high-seismic regions are helpful, they are not sufficient for blast design. Because of the localized nature of an explosion, such provisions must be followed even for elements that are not part of the lateral-force-resisting system, especially on the exterior. In case a primary supporting element does fail because of a blast, the structural system should include alternate load paths so that progressive collapse of additional bays will not follow. Multistory buildings are especially vulnerable in this respect and should have enough inherent redundancy to survive a local failure, especially at the ground floor level.

Elements must have adequate shear capacity to ensure that they do not fail in this mode while responding in flexure. It is usually sufficient to design for ultimate static design values of V_u equal to the reactions produced by a uniformly distributed load with a total magnitude of R_n or $2F_o$, whichever is smaller. Supporting elements can then be conservatively designed to have ultimate strengths adequate to resist V_u. Concrete and masonry elements must have appropriate reinforcement at and near supports, especially when $\theta > 2°$ or V_u exceeds the flexural or direct shear capacity V_n of the base material. At the face of a support where there is a construction joint or V_u exceeds the direct shear capacity, adequate shear friction reinforcement is required.

SUMMARY

The following steps constitute an equivalent static approach for blast effects analysis and design:

1. Determine standoff distance R and explosive charge weight W.
2. Determine peak reflected pressure p_r and effective load duration t_d from scaled distance $Z = R/W^{1/3}$.
3. Calculate effective mass m_e, effective stiffness k_e, natural period of vibration T, and ultimate strength R_n from Tables 1 and 2 and Equation (1).
4. Select level of protection from Table 3, acceptable damage from Table 4, and associated ductility ratio and end rotation limits μ_{max} and θ_{max} from Table 5 or Table 6.
5. Calculate response modification factor R_b and ultimate load F_u from Equations (2), (3), and (6).
6. Verify $F_u \leq R_n$; if not, revise element properties and return to step 3.
7. Verify $V_u \leq V_n$; if not, provide adequate reinforcement.
8. Employ proper detailing that will provide the necessary ductility and shear capacity.

This procedure is valid when the postulated threat is exterior to the structure of interest and consists of a small to medium explosive at a modest distance or a large explosive at a considerable distance ($Z \geq 3$), with few additional reflecting surfaces nearby. More in-depth evaluation is necessary for special situations such as close-in and confined explosions and when surrounding features could significantly alter the blast environment.

NOTATION

The following symbols are used in this paper:

E	=	modulus of elasticity;
F_o	=	maximum dynamic load;
F_u	=	ultimate static design load;
F_y	=	yield strength of steel;
I	=	moment of inertia;
i_r	=	total reflected impulse;
k_1	=	elastic stiffness prior to formation of the first plastic hinge;
k_2	=	elastic-plastic stiffness between formation of the first plastic hinge and formation of a mechanism;
k_e	=	effective stiffness;
L	=	span length;
M_{nre}	=	remaining nominal moment capacity at a fixed end;
M_{nrm}	=	remaining nominal moment capacity at mid-span;
m	=	uniformly distributed mass;
m_e	=	effective lumped mass;
p_r	=	peak reflected pressure;
R	=	standoff distance;
R_b	=	response factor for converting maximum dynamic load to ultimate static design load;
R_n	=	ultimate strength upon formation of a mechanism;
R_y	=	yield strength upon formation of the first plastic hinge;
T	=	natural period of vibration;
t_d	=	design duration of the positive phase of blast loading;
V_n	=	flexural or direct shear capacity;
V_u	=	ultimate static design shear;
W	=	explosive charge size in equivalent weight or mass of TNT;
y_e	=	effective yield deflection corresponding to ultimate strength and effective stiffness;
y_{max}	=	maximum blast deflection;
Z	=	scaled distance for determining blast loading;
μ	=	ductility ratio;
μ_{cr}	=	limiting ductility ratio accounting for maximum acceptable end rotation;
μ_{max}	=	maximum acceptable ductility ratio;
θ	=	end rotation at a support;
θ_{max}	=	maximum acceptable end rotation at a support;
ω_p	=	reinforcement index for a prestressed concrete element;

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REFERENCES

ASCE (1997). *Design of Blast Resistant Buildings in Petrochemical Facilities*. Task Committee on Blast Resistant Design, ASCE, Reston, Va.

ASCE (1999). *Structural Design for Physical Security: State of the Practice*. Task Committee, ASCE, Reston, Va.

ASCE (2005). *Minimum Design Loads for Buildings and Other Structures*. ASCE, Reston, Va.

Biggs, J. M. (1964). *Introduction to Structural Dynamics*. McGraw-Hill, New York, N.Y.

Department of Defense (2007). *DoD Minimum Antiterrorism Standards for Buildings*. UFC 4-010-01.

Departments of the Army, the Navy, and the Air Force (1990). *Structures to Resist the Effects of Accidental Explosions*. TM 5-1300 / NAVFAC P-397 / AFR 88-22.

Newmark, N. M. (1956). "An Engineering Approach to Blast Resistant Design." *Trans. ASCE*, Vol. 121, pp. 45-64.

Protective Design Center (2006). *Single Degree of Freedom Response Limits for Antiterrorism Design*. PDC-TR 06-08.